100 YEARS OF GENERAL RELATIVITY
WHAT ARE WE CELEBRATING?

Hanoch Gutfreund

THE HEBREW UNIVERSITY OF JERUSALEM

Albert Einstein.™

A CENTURY OF GENERAL RELATIVITY
November 30 – December 2, 2015 | Harnack House Berlin

The Drama of November (1915) – four papers submitted to the Royal Prussian Academy of Science

"*On the General Theory of Relativity*", Nov. 4th

"…fatal prejudice", "…key to the solution"

"*On the General Theory of Relativity (Addendum)*", Nov. 11th

"…an even more concise and logical structure"

"*Explanation of Perihelion Motion of Mercury from the General Theory of Relativity*", Nov. 18th

"…an important confirmation of this most fundamental theory.."

"*The Field Equations of Gravitation*", Nov. 25th

"…finally completed the general theory of relativity as a logical structure."

Einstein or Hilbert – who got there first?

"The differential equations of gravitation that result here are, as it seems to me, in agreement with the magnificent theory of general relativity established by Einstein."

D. Hilbert, in "On the Foundations of Physics (First Contribution)."

"On the General Theory of Relativity", Nov. 4th

- It turned out that a slight adjustment of the expression for the gravitational field itself was sufficient to turn the theory developed and published with Marcel Grossmann (the *Entwurf* theory) to obtain the so-called "November theory."

- The new expression for the gravitational field is the Christoffel symbol, which is a combination of derivatives of the components of the metric tensor

- Einstein refers to the former version of the gravitational field as a "fatal prejudice," and a few weeks later, in a letter to Arnold Sommerfeld, he described the identification of the gravitational field with the Christoffel symbol as "the key to the solution."

Einstein was fascinated by the power of mathematical formalism to lead to the correct theory: *"Nobody who really grasped it can escape from its charm, because it signifies a real triumph of the general differential calculus as founded by GAUSS, RIEMANN, CHRISTOFFEL, RICCI AND LEVI-CIVITA."* (The names are capitalized in the original publication.)

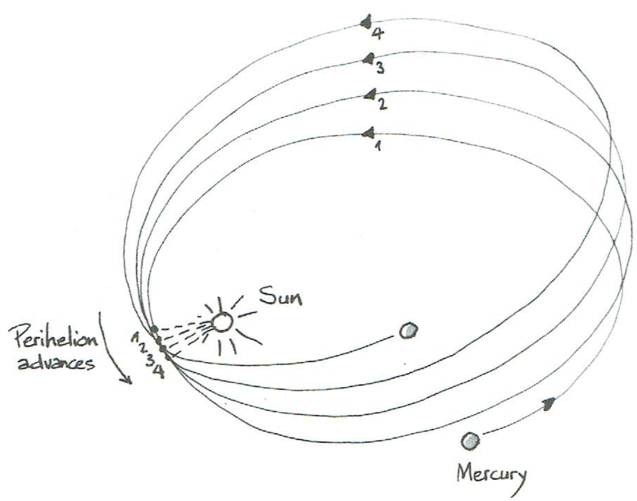
"On the General Theory of Relativity (Addendum)", Nov. 11th

- *"I now want to show here that an even more concise and logical structure of the theory can be achieved by introducing an admittedly bold additional hypothesis on the structure of matter."*

- Einstein thus proposed a new interpretation of the formalism of the November theory. If one assumes that the only fields occurring as sources of gravitation are electromagnetic fields, so that ultimately all matter can be reduced to the latter, then a field equation based on the Ricci tensor can be formulated without imposing any further coordinate restriction.

- If one introduces this bold assumption, the conservation principle no longer implies any constraint on the admissible coordinate systems, but instead becomes a stipulation on admissible sources of the gravitational field. By considering (temporarily) a speculative hypothesis on the nature of matter, Einstein moved one step closer to his goal of a generally covariant theory.

“*Explanation of Perihelion Motion of Mercury from the General Theory of Relativity*”, Nov. 18th



5600” per century

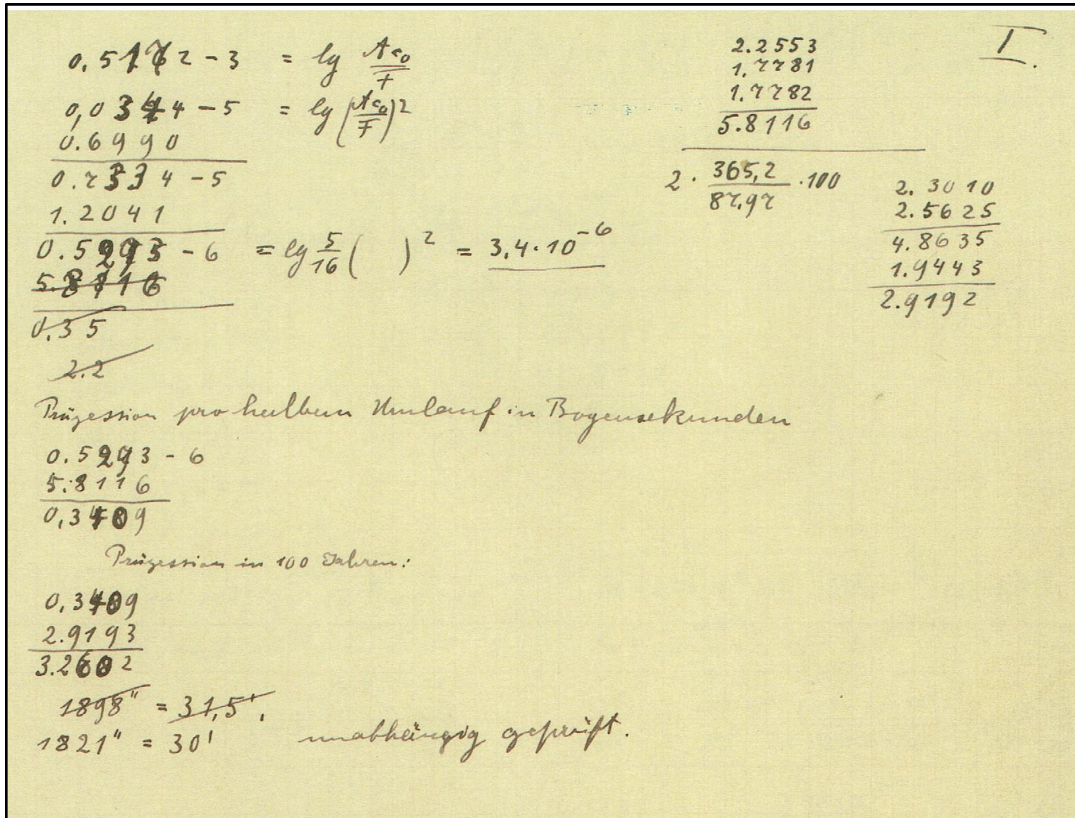
5557” explained by Newton’s theory of the solar system

43” remained unexplained

Already in 1907 Einstein expected that the new theory will explain this anomaly

Einstein – Besso Manuscript

Calculation of the Mercury perihelion precession



I.

$0,5172 - 3 = \lg \frac{A c_0}{F}$

$0,0344 - 5 = \lg \left(\frac{A c_0}{F}\right)^2$

0.6990

$0.7334 - 5$

1.2041

$0.5293 - 6 = \lg \frac{5}{16} (\quad)^2 = 3,4 \cdot 10^{-6}$

5.8116

$0,35$

2.2

2.2553

1.7781

1.7782

5.8116

$2 \cdot \frac{365,2}{87,97} \cdot 100$

2.3010

2.5625

4.8635

1.9443

2.9192

Präzession pro halben Umlauf in Bogensekunden

$0.5293 - 6$

5.8116

$0,3409$

Präzession in 100 Jahren:

$0,3409$

2.9193

3.2602

~~1898" = 31,5'~~

1821" = 30' unabhängig geprüft.

EB manuscript, p. 28

Einstein records the final result for the "precession in 100 years" of Mercury's perihelion produced by the field equations of the Einstein-Grossmann theory. This theory predicts about 18" per century, but the result achieved here, 1821" = 30', which Einstein claims was "independently checked", is a factor of 100 too large. Einstein almost certainly realized that this result was off by a factor of 100 but it was Besso who found the source of the error.

"Most gratifying is the agreement with perihelion motion and the general covariance; strangest, however, is the circumstance that Newton's theory of the field is incorrect already in the first order. It is just the circumstance that the g_{11}, g_{22}, g_{33} (components of the metric tensor) do not appear in first-order approximation of the equation of motion which determines the simplicity of Newton's theory." Einstein to Besso, Dec. 1915

- **For Einstein, the requirement that an acceptable theory of gravitation would reduce to the Newtonian theory as a limiting or special case was not only natural, but absolutely essential.**
- **In the course of the Einstein-Grossman collaboration this requirement served not only as a condition for an acceptable gravitational field equation, but also as a starting point for its construction.**
- **In 1912, they rejected the Ricci tensor, a natural candidate for the gravitation tensor in the source free case, because they thought (erroneously) that it did not reduce to the Newtonian expression in the case of an infinitely weak field.**

"The Field Equations of Gravitation", Nov. 25th

- All that was required was to change the way in which the sources of the gravitational field were represented the right-hand-side of the field equation.

- If an additional term, namely the trace of the energy-momentum tensor is added to the source term then all the additional conditions become superfluous. In particular, the conservation principle is also satisfied as an automatic consequence of the modified field equation.

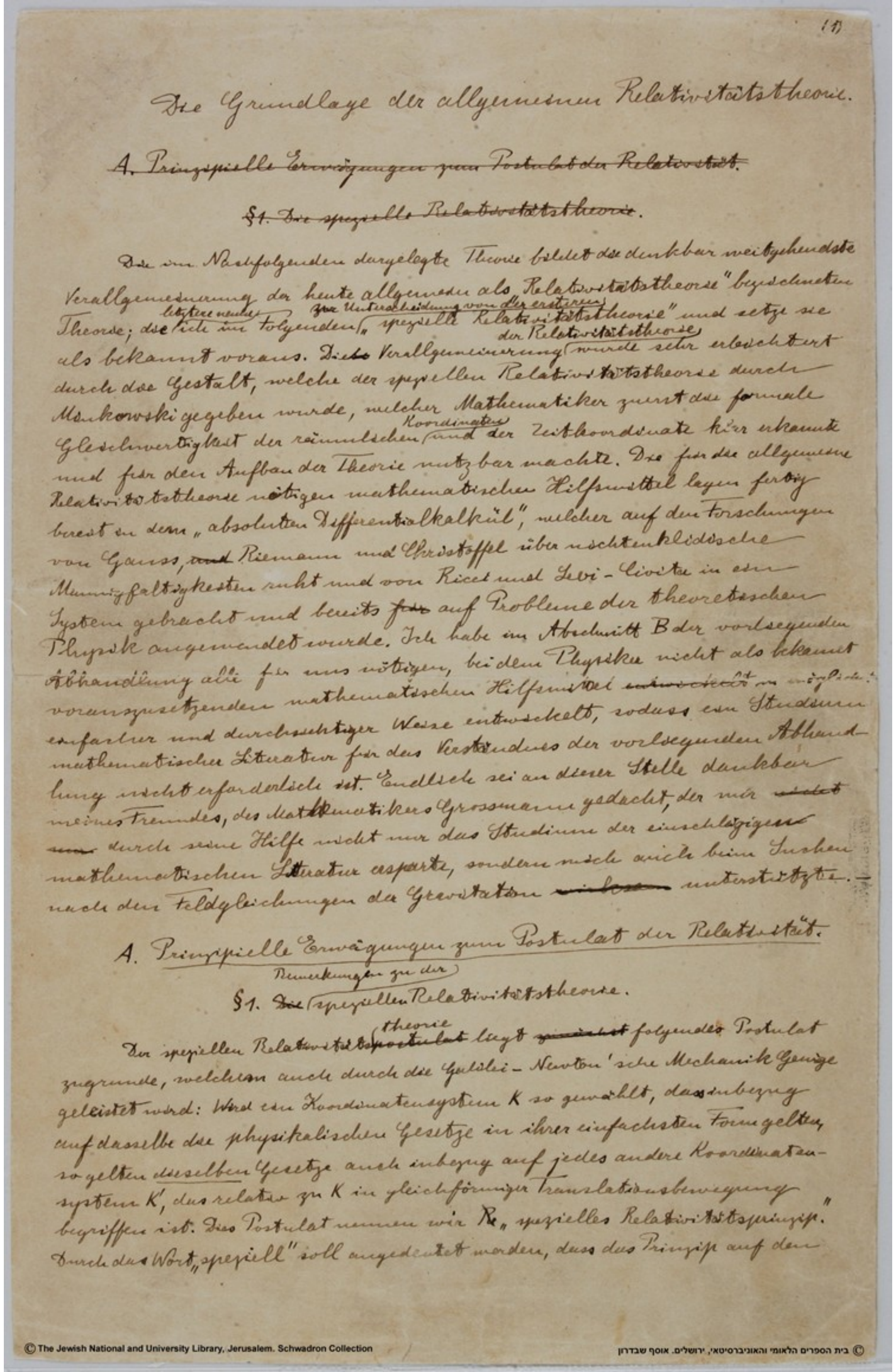
- Alternatively, also the left-hand-side can be changed accordingly, so that instead of the Ricci tensor, the modified expression now known as the Einstein tensor, constitutes this left-hand-side.

- This could be done only after Einstein realized how the Newtonian limit should have been interpreted.

What motivated Einstein to launch the search for GR?
On what knowledge was it built?
No empirical/observational motivation

Since the introduction of the special principle of relativity has been justified, every intellect which strives after generalization must feel the temptation to venture the step towards the general principle of relativity.
Einstein, *Relativity, Special and General Theory (a popular account), 1917*

"I have learned something else from the theory of gravitation: no collection of empirical facts, no matter how comprehensive, can ever lead to the formulation of such complicated equations … [they] can only be found through the discovery of a logically simple mathematical condition that completely or almost completely determines the equations. Once one has those sufficiently strong formal conditions, one requires only little knowledge of facts to set up a theory."
Einstein, *Autobiographic Notes, 1949*



(1)

Die Grundlage der allgemeinen Relativitätstheorie.

~~A. Prinzipielle Erwägungen zum Postulat der Relativität.~~

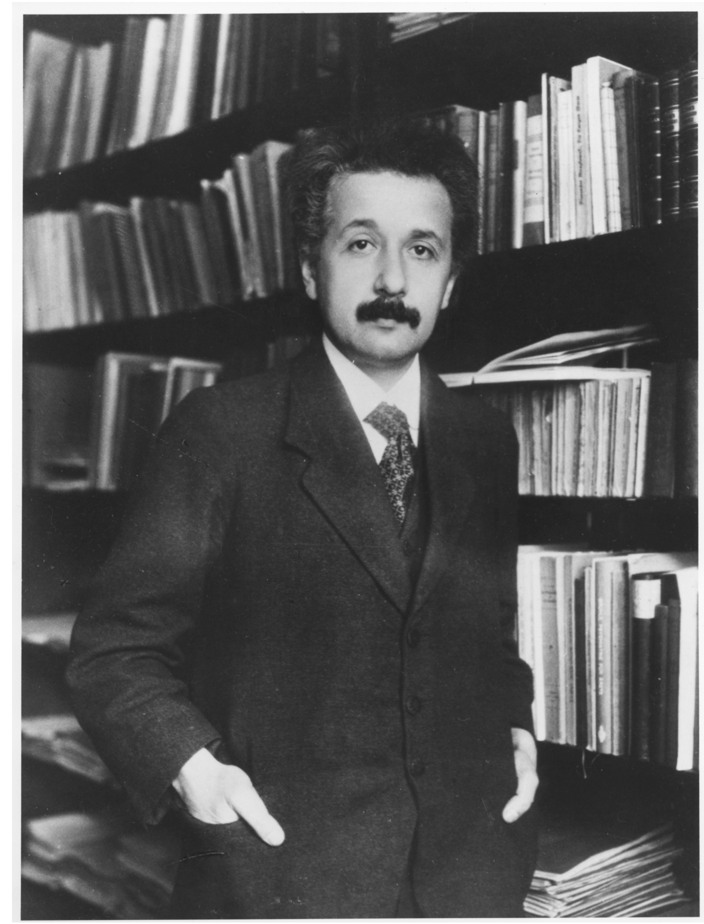
~~§1. Die spezielle Relativitätstheorie.~~

Die im Nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im Folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathematiker zuerst die formale Gleichwertigkeit der räumlichen Koordinaten und der Zeitkoordinate klar erkannte und für den Aufbau der Theorie nutzbar machte. Die für die allgemeine Relativitätstheorie nötigen mathematischen Hilfsmittel lagen fertig bereit in dem „absoluten Differentialkalkül", welcher auf den Forschungen von Gauss, ~~und~~ Riemann und Christoffel über nichteuklidische Mannigfaltigkeiten ruht und von Ricci und Levi-Civita in ein System gebracht und bereits ~~für~~ auf Probleme der theoretischen Physik angewendet wurde. Ich habe im Abschnitt B der vorliegenden Abhandlung alle für uns nötigen, bei dem Physiker nicht als bekannt vorauszusetzenden mathematischen Hilfsmittel in möglichst einfacher und durchsichtiger Weise entwickelt, sodass ein Studium mathematischer Literatur für das Verständnis der vorliegenden Abhandlung nicht erforderlich ist. Endlich sei an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichungen der Gravitation unterstützte.

A. Prinzipielle Erwägungen zum Postulat der Relativität.

§1. ~~Die~~ Bemerkungen zu der speziellen Relativitätstheorie.

Der speziellen Relativitätstheorie liegt folgendes Postulat zugrunde, welchem auch durch die Galilei-Newton'sche Mechanik Genüge geleistet wird: Wird ein Koordinatensystem K so gewählt, dass in bezug auf dasselbe die physikalischen Gesetze in ihrer einfachsten Form gelten, so gelten dieselben Gesetze auch in bezug auf jedes andere Koordinatensystem K', das relativ zu K in gleichförmiger Translationsbewegung begriffen ist. Dies Postulat nennen wir „spezielles Relativitätsprinzip". Durch das Wort „speziell" soll angedeutet werden, dass das Prinzip auf den



Einstein in 1916

Die Ausrechnung ergibt

$$B = \frac{2\alpha}{\Delta} = \frac{\kappa M}{4\pi\Delta} \quad \ldots\ldots (74)$$

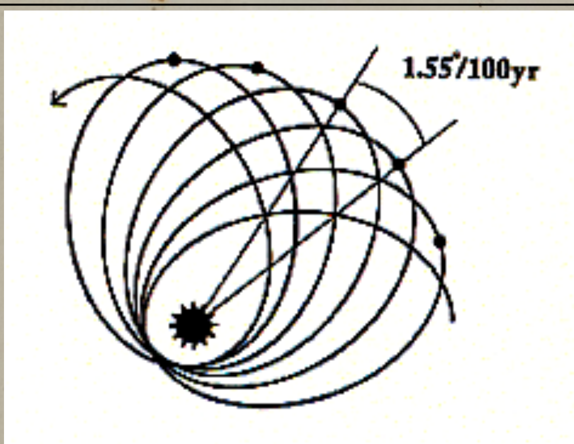
Ein an der Sonne vorbeigehender Lichtstrahl erfährt demnach eine Biegung von 1,7", ein am Planeten Jupiter vorbeigehender eine solche von etwa 0,02".

Berechnet man das Gravitationsfeld um eine Grössenordnung genauer und ebenso mit entsprechender Genauigkeit die Bahnbewegung eines materiellen Punktes von relativ unendlich kleiner Masse, so erhält man gegenüber den Kepler-Newton'schen Gesetzen der Planetenbewegung eine Abweichung von folgender Art. Die ~~grosse Achse~~ Bahnellipse eines Planeten erfährt in Richtung der Bahnbewegung ~~pro Umlauf~~ eine langsame Drehung, ~~welche pro~~ vom Betrage

$$\varepsilon = 24\pi^3 \frac{a^2}{T^2 c^2 (1-e^2)} \quad \ldots\ldots (75)$$

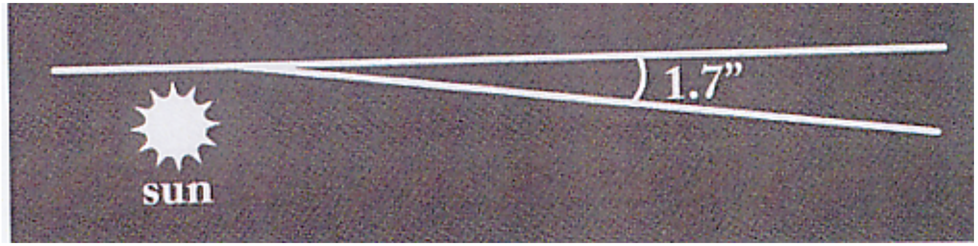
pro Umlauf. In dieser Formel bedeutet a die grosse Halbachse, c die Lichtgeschwindigkeit in üblichem ~~Masse~~ Maße, e die Exzentrizität, T die Umlaufszeit in Sekunden.

Die Rechnung ergibt für den Planeten Merkur ~~eine~~ eine Drehung der Bahn ~~um~~ um 43" pro Jahrhundert, genau entsprechend der Konstatierung der Astronomen (Leverrier); ~~welche~~ diese fanden nämlich einen durch Störungen der übrigen Planeten nicht erklärbaren Rest der Perihelbewegung dieses Planeten von der angegebenen Grösse.



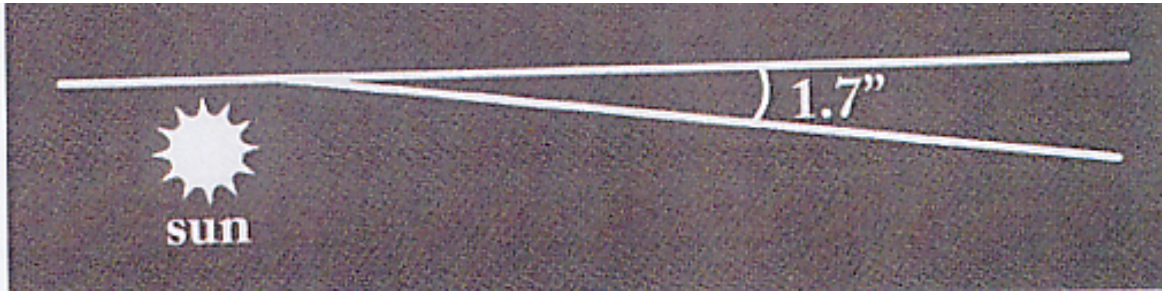
Bezüglich der Rechnung verweise ich auf die Originalabhandlungen
A. Einstein Sitz. Ber. d. Preuss. Akad. d. W. XLVII. 1915. S. 831
K. Schwarzschild, Sitz. Ber. d. Preuss. Akad. d. W. VII. 1916. S. 189.

According to this, a ray of light going past the sun undergoes a deflection of 1.7"…



Calculation gives for the planet Mercury a rotation of the orbit of 43" per century, corresponding exactly to astronomical observations (Leverrier); for astronomers have discovered in the motion of perihelion of this planet, after allowing for disturbances by other planets, an inexplicable remainder of this magnitude.

When Einstein saw this result, he was so excited that, as he told one of his former collaborators, he had heart palpitations.



REVOLUTION IN SCIENCE.

NEW THEORY OF THE UNIVERSE.

NEWTONIAN IDEAS OVERTHROWN.

Yesterday afternoon in the rooms of the Royal Society, at a joint session of the Royal and Astronomical Societies, the results obtained by British observers of the total solar eclipse of May 29 were discussed.

The London Times
7. 11. 1919

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

The New York Times

The end (?)

100 YEARS OF GENERAL RELATIVITY

- **“The Formative years” (H.G. and Jurgen Renn)**
- **“The Low-water mark period” (Jean Eisenstadt)**
- **“The Renaissance” (Clifford Will)**
- **“The Golden Age” (Kip Thorne)**

100 YEARS OF GENERAL RELATIVITY

"The Formative years" (H.G. and Jurgen Renn) from 1915 roughly until the late 1920s and early 1930s, when more and more of the leading physicists turned their attention to the booming field of quantum mechanics to debate its meaning and explore its consequences.

"The Low-water mark period" (Jean Eisenstadt) immediately following it, in which, also due to the WWII, faded into the background, being largely considered as irrelevant to mainstream physics and limited to the explanation of some minor adjustments of Newton's otherwise well-confirmed theory of gravitation.

"The Renaissance" (Clifford Will) of relativity begins after the war and makes its prominent mark only more than a decade later in connection with new astrophysical discoveries in the 1960s such as those of quasars.

"The Golden Age" (Kip Thorne) of relativity bringing new conceptual insights such as those into the nature of black holes and turning the theory into the foundation of modern astrophysics and observational cosmology.

The Formative Years

Einstein's Intellectual Odyssey to General Relativity did not end with the formulation of the field equation in the concluding paper of November 1915. The tensions between Einstein's heuristics and the implications of the new theory also characterized the further evolution until at least the early 1930s.

Immediately after its creation, general relativity became a hallmark of international cooperation in the world of science at a time when the Great War was dividing it. Einstein's theory was taken up, elaborated, and controversially discussed by his colleagues - physicists, mathematicians, astronomers, and philosophers.

Einstein himself made further fundamental contributions to the development of his theory, exploring consequences such as gravitational waves and cosmological solutions, clarifying concepts such as that of the conservation of energy and momentum, and even reinterpreting basic aspects of the theory.

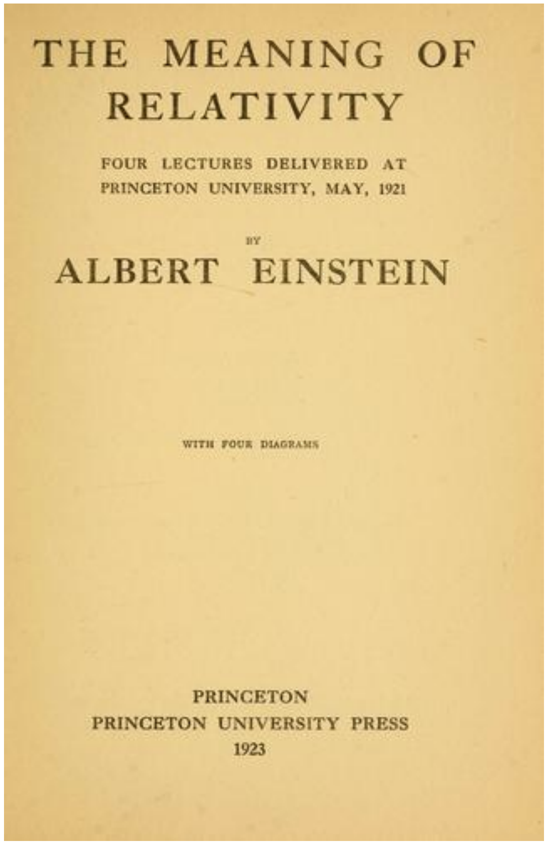
The Mathematical Formulation of the Theory

"My series of gravitation papers are a chain of wrong tracks, which nevertheless did gradually lead closer to the objectives. That is why now finally the basic formulas are good, but the derivations abominable; this deficiency must still be eliminated."
Einstein to Lorentz, November 1915

In his original formulation of general relativity, Einstein did not interpret the theory in terms of differential geometry. When he discusses the Riemann-Christoffel tensor, for instance, he does not even mention curvature. The affine connection or parallel transport are not mentioned.

The geometrization of general relativity and the understanding of gravity as being due to the curvature of spacetime is a result of the further development and not a presupposition of Einstein's formulation of the theory.

The theory, as we know it today is first presented in Einstein's *The Meaning Relativity* also known as the *Four Lectures on Relativity*.



The *Meaning of Relativity* is the paradigmatic text of the *formative years* of general relativity in which the theory essentially received the form in which it became one of the pillars of modern physics.

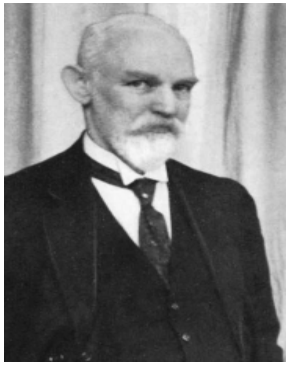
Einstein evidently returned to the theory of relativity in many later publications, but he never made another attempt at such a complete presentation in which he explained, and discussed its basic principles and their consequences.

“Cosmological Considerations in the General Theory of Relativity”, February 1917

Einstein-deSitter dialogue and controversy (started in 1916)

- *“In a consistent theory of relativity, there can be no inertia relative to ‘space’, but only an inertia of masses relative to one another.”*
- *“… for the system of the fixed stars no boundary conditions of the kind can come into question at all, as was also rightly emphasized by the astronomer de Sitter recently.”*
- A static, closed universe

THE EINSTEIN – DE SITTER – WEYL – KLEIN DEBATE (1918-9)



de Sitter, Willem (1872–1934)

Weyl, Hermann (1885–1955)

Klein, Felix (1849-1925)

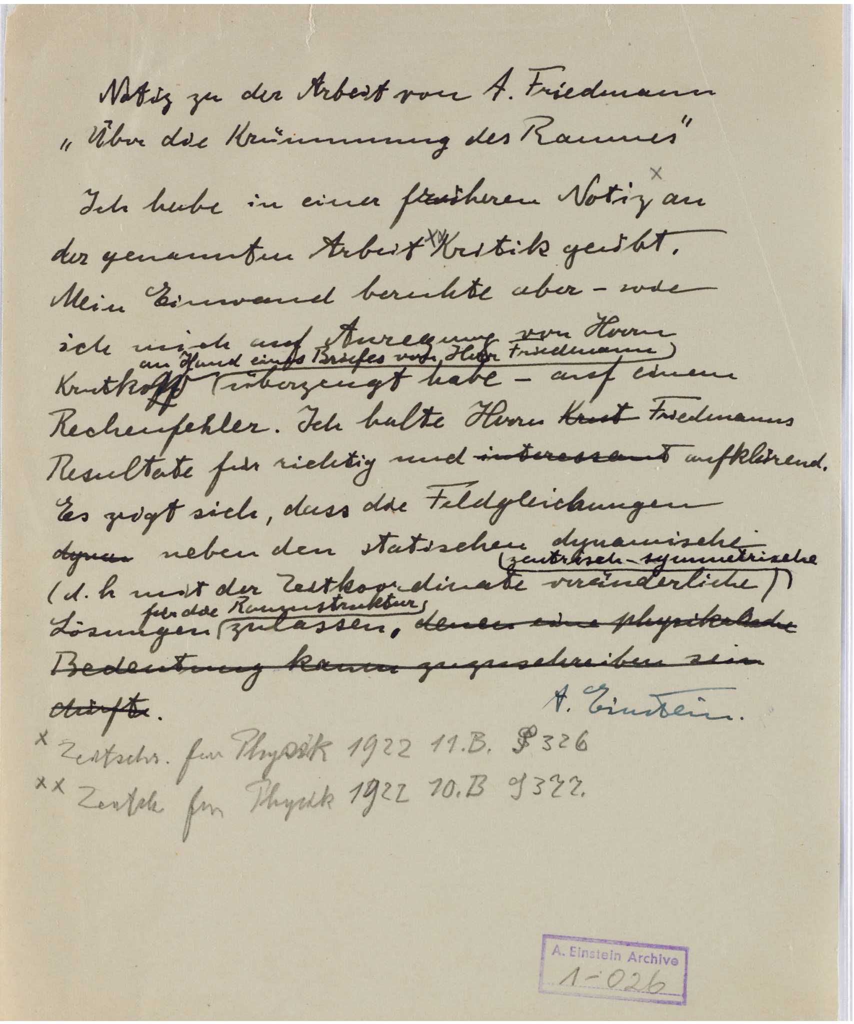
- Einstein's static closed universe
- The cosmological constant
- deSitter criticism
- Mach's principle

“ Mach’s Principle…

…the g-field is completely determined by the masses of the bodies. …….The necessity to uphold it is by no means shared by all colleagues; but I myself feel it is absolutely necessary to satisfy it. With , according to the field equations of gravitation, there can be no g-field without matter. Obviously, this postulate is closely connected to the space-time structure of the world as a whole, because all masses in the universe will partake in the generation of the g-field.”

In this way Einstein translated Mach’s ideas from the language of mechanics to that of field theory.

Friedman’s Expanding Universe Model



Notiz zu der Arbeit von A. Friedmann „Über die Krümmung des Raumes“

Ich habe in einer früheren Notiz[x] an der genannten Arbeit[xx] Kritik geübt. Mein Einwand beruhte aber – wie ich mich auf Anregung von Herrn Krutkoff (an Hand eines Briefes von Herrn Friedmann) überzeugt habe – auf einem Rechenfehler. Ich halte Herrn ~~Krut~~ Friedmanns Resultate für richtig und ~~interessant~~ aufklärend. Es zeigt sich, dass die Feldgleichungen ~~dyn~~ neben den statischen dynamische (d. h. mit der Zeitkoordinate veränderliche) (zentrisch-symmetrische) Lösungen für die Raumstruktur zulassen, ~~denen eine physikalische Bedeutung kaum zuzuschreiben sein dürfte~~.

A. Einstein.

x Zeitschr. für Physik 1922 11.B. S 326

xx Zeitschr. für Physik 1922 10.B S 322.

Note on A. Friedmann’s work, ‘On the curvature of space’

“In my previous work I have criticized the cited work, but my objection……rested on an error in my calculations. I consider that Mr. Friedmann’s results are correct and shed new light.

It follows that the field equations, besides the static solution, permit dynamic (that is varying with time coordinate) spherically symmetric solutions for the spatial structure”.

He added the words: ***“but a physical significance can hardly be ascribed to them”,*** which he crossed out before sending the note to the editor.

Friedmann, Alexander Alexandrowitsch (1888–1925)

Hubble, Edwin (1889–1953)

Lemaître, Georges (1894–1966)

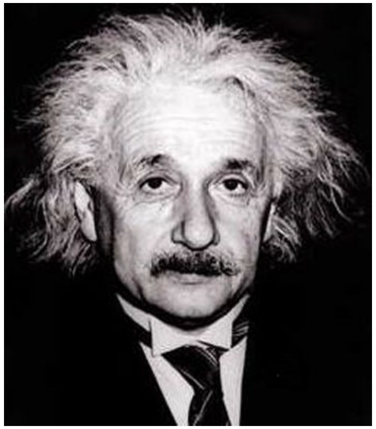
Meaning of Relativity, 2nd edition (1945), Appendix

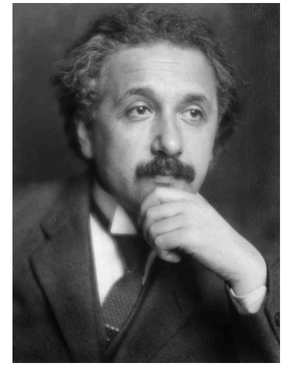
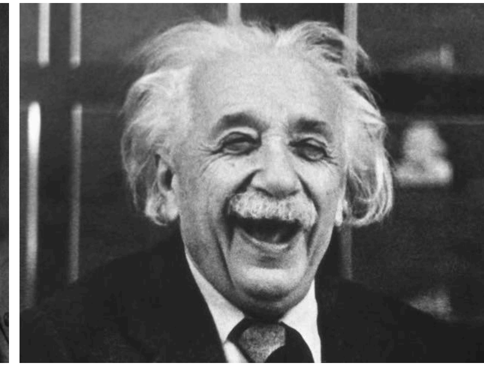
Rederivation of Friedmann's model
Long and detailed discussion of Hubble's observation
The problem of a too young universe

"If Hubble's expansion had been discovered at the time of the creation of the general theory of relativity, the cosmologic member would never have been introduced."

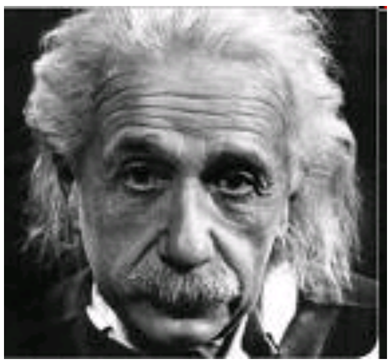
For Einstein's Machian philosophical conception of a static universe, the cosmological constant was a lifesaver. Eventually, however, he was forced to realize that this constant did not accomplish the purpose for which it had been invented and he abandoned it. But the additional term is really quite legitimate and contrary to what Einstein wrote in 1916, the resulting field equation was actually not the most general equation consistent with his demands.

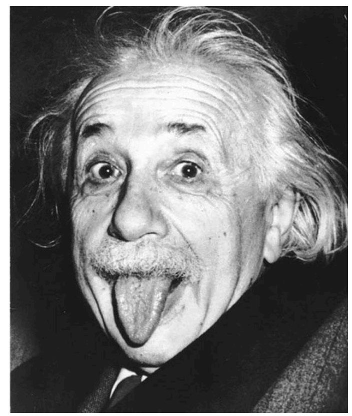
A very inappropriate moment to abandon the cosmological constant. Had he kept it he could have adapted it to fix the age of the universe problem.

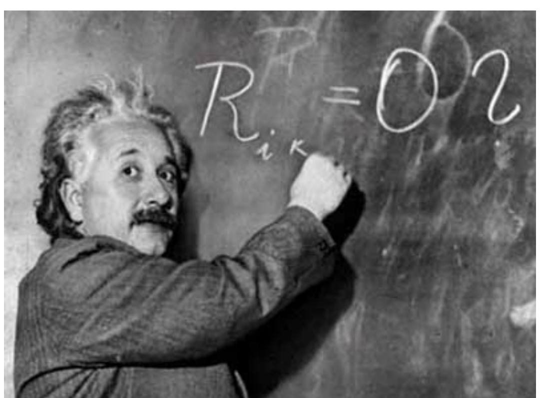
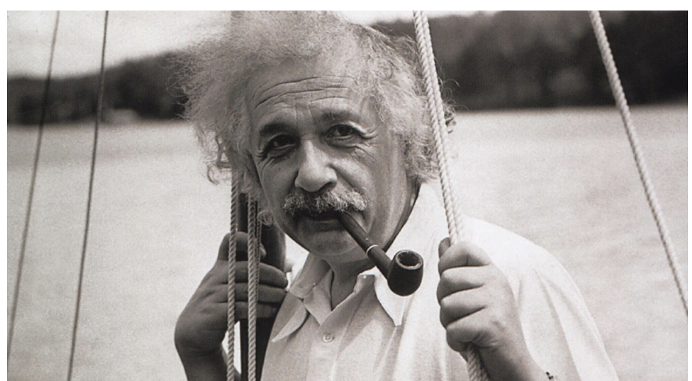


WHAT ARE WE CELEBRATING?





$$
\begin{aligned}
G^{\mu\nu} = \sum_{i\kappa}\left(g^{\mu i}g^{\nu\kappa} - \frac{1}{2}g^{\mu\nu}g^{i\kappa}\right)
&\left\{\frac{1}{2}g^{lm}\left(\frac{\partial^2 g_{im}}{\partial x^\kappa \partial x^l} + \frac{\partial^2 g_{\kappa m}}{\partial x^i \partial x^l} - \frac{\partial^2 g_{i\kappa}}{\partial x^m \partial x^l} - \frac{\partial^2 g_{lm}}{\partial x^i \partial x^\kappa}\right)\right. \\
&+ \frac{1}{2}\frac{\partial g^{lm}}{\partial x^l}\left(\frac{\partial g_{im}}{\partial x^\kappa} + \frac{\partial g_{\kappa m}}{\partial x^i} - \frac{\partial g_{i\kappa}}{\partial x^m}\right) - \frac{1}{2}\frac{\partial g^{lm}}{\partial x^\kappa}\frac{\partial g_{lm}}{\partial x^i} \\
&- \frac{1}{4}g^{jm}g^{ln}\left[\left(\frac{\partial g_{ml}}{\partial x^i} + \frac{\partial g_{im}}{\partial x^l} - \frac{\partial g_{li}}{\partial x^m}\right)\left(\frac{\partial g_{jn}}{\partial x^\kappa} + \frac{\partial g_{\kappa n}}{\partial x^j} - \frac{\partial g_{\kappa j}}{\partial x^n}\right)\right. \\
&\left.\left. - \left(\frac{\partial g_{im}}{\partial x^\kappa} + \frac{\partial g_{\kappa m}}{\partial x^j} - \frac{\partial g_{i\kappa}}{\partial x^m}\right)\frac{\partial g_{ln}}{\partial x^j}\right]\right\} = \bar{\kappa}T^{\mu\nu}
\end{aligned}
$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \bar{\kappa}T^{\mu\nu}$$

$$T^{\mu\nu} = \rho\frac{dx^\mu}{ds}\frac{dx^\nu}{ds},$$